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LETTER TO THE EDITOR

Electric current fluctuations in extended irreversible thermodynamics

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Abstract. Starting from the generalised Gibbs equation of extended irreversible thermodynamics, a thermodynamic potential which offers a suitable description of the electric current fluctuations in an isotropic rigid conductor is derived.

The limitations of the local equilibrium hypothesis have led some authors (Müller 1967, Lambermont and Lebon 1973, Lebon 1978, Jou *et al* 1979 and Lebon *et al* 1979) to propose a generalised Gibbs equation which takes into account the dependence of a non-equilibrium entropy on the dissipative fluxes, as well as on the classical thermodynamic variables.

Dealing with a generalised Gibbs equation for rigid heat conductors (Lambermont and Lebon 1973), Jou and Rubí (1979) calculated the heat flux fluctuations in the framework of the extended irreversible thermodynamics (Lebon *et al* 1979). Our purpose in this Letter is to reformulate in a more precise way the above development and to apply it to the treatment of electric current fluctuations in an isotropic rigid conductor. Our formalism leads in a direct way to the classical results of the Nyquist theorem (Landau and Liftshitz 1967).

Classically, the state of the rigid conductor in the presence of a steady external electric field $\boldsymbol{E}$ is specified by the local values of the internal energy density u and the electron density c_e, while the electric current $\boldsymbol{J}_e$ is expressed as a function of the external field and of the gradient of these quantities. Here, on the contrary, $\boldsymbol{J}_e$ is taken as an independent variable and therefore we assume that the thermodynamic state is specified by the local values of u, c_e and $\boldsymbol{J}_e$. For simplicity, the heat flux has been neglected. In the framework of extended irreversible thermodynamics, we assume the existence of a non-equilibrium entropy s, whose dependence on u, c_e and $\boldsymbol{J}_e$ is described by the following generalised Gibbs equation,

$$\mathrm{d}s = T^{-1}\,\mathrm{d}u - T^{-1}\mu_e\,\mathrm{d}c_e + T^{-1}\rho^{-1}\gamma \boldsymbol{J}_e\,\mathrm{d}\boldsymbol{J}_e, \tag{1}$$

where ρ is the density and T, μ_e and γ, which are given by the subsequent equation of state of (1), are respectively the absolute temperature, the chemical potential of the electrons and a coefficient which will be determined.

A similar generalised Gibbs equation has been justified from the microscopic basis of the kinetic theory of gases by Müller (1967). Also, such an equation has been given a kinetic justification in the Chapman–Enskog development by Lebon (1978) and in the thirteen-moments theory of Grad by Jou *et al* (1979).

While the time evolution of the variables u and c_e is given by the well known balance equations of energy and electron density, respectively

$$\rho \dot{u} = \boldsymbol{J}_e \cdot \boldsymbol{E}, \qquad \rho \dot{c}_e = -\nabla \cdot \boldsymbol{J}_e, \tag{2}$$

the time evolution of $\boldsymbol{J}_e$ remains to be obtained. In order to get it, we derive the entropy balance from (1) and (2), which lead to the expression

$$\rho \dot{s} + \nabla \cdot \boldsymbol{J}_s = \sigma_s \tag{3}$$

where $\boldsymbol{J}_s$, the entropy flux, and σ_s, the entropy production, are given respectively by

$$\boldsymbol{J}_s \equiv \mu_e T^{-1} \boldsymbol{J}_e$$

and (4)

$$\sigma_s \equiv T^{-1} \boldsymbol{J}_e \cdot [\boldsymbol{E} + \gamma \dot{\boldsymbol{J}}_e - T \cdot \nabla(\mu_e/T)].$$

We assume for simplicity that μ_e and T are constant over the system. In view of the bilinear character of (4), the simplest equation for $\dot{\boldsymbol{J}}_e$ compatible with the positive character of σ_s required by the second law can be obtained by writing $\boldsymbol{E} + \gamma \dot{\boldsymbol{J}}_e$ as a function of $\boldsymbol{J}_e$. This leads to

$$\dot{\boldsymbol{J}}_e = -\tau^{-1}(\boldsymbol{J}_e - \sigma \boldsymbol{E}) \tag{5}$$

where τ, the electric current relaxation time, is related to γ by $\tau = -\gamma\sigma$ with σ the electrical conductivity, which must be positive in view of the second law. Phenomenological laws of the above type have been frequently used in magneto-hydrodynamics (Spitzer 1956). The need of phenomenological laws like (5) depends on the values of τ. Note that when τ tends to zero the classical Ohm's law is recovered.

We have then obtained an identification of the coefficient γ in terms of physically defined quantities, and this allows us to write (1) in the form

$$\mathrm{d}s = T^{-1}\mathrm{d}u - T^{-1}\mu_e \mathrm{d}c_e - \rho^{-1}(\tau/\sigma T)\boldsymbol{J}_e \cdot \mathrm{d}\boldsymbol{J}_e \tag{6}$$

whose integrated form gives

$$\rho s = \rho s_{\mathrm{eq}} - (\tau/2\sigma T)\boldsymbol{J}_e^2. \tag{7}$$

We define the function ϕ as the purely non-equilibrium part of ρs, i.e. as

$$\phi \equiv \rho s - \rho s_{\mathrm{eq}} \tag{8}$$

which, if the local equilibrium state is assumed to be stable, satisfies $\delta^2\phi \leq 0$. Jou and Rubí (1979) observed that the curvature of ϕ is indeed related to the heat flux fluctuations. Here we state the hypothesis that the probability $P_r(\delta \boldsymbol{J}_e)$, of a fluctuation $\delta \boldsymbol{J}_e = \boldsymbol{J}_e - \sigma \boldsymbol{E}$ of the electric current with respect to its steady state value $\sigma \boldsymbol{E}$, is given by

$$P_r(\delta \boldsymbol{J}_e) \sim \exp(\delta^2\phi/2k) \tag{9}$$

where k is the Boltzmann constant, in analogy with the well known Einstein relation for equilibrium fluctuations. We note that this is not in fact the Einstein hypothesis, since we are not developing ϕ near a maximum. Rather, this approach has some similarity with that proposed by some authors (Jähnig and Richter 1976, Keizer 1976) which describes the fluctuations through a relation of the kind (9). Here, on the contrary, we show that the potential ϕ constructed *a priori*, is able to describe the fluctuations of the electric current. This leads to

$$P_r(\delta \boldsymbol{J}_e) \sim \exp[-(\tau/2\sigma k T)\delta \boldsymbol{J}_e^2]. \tag{10}$$

The subsequent correlation function for $\delta \boldsymbol{J}_e$ is then

$$\langle \delta \boldsymbol{J}_{e_i}(\boldsymbol{r}, t) \delta \boldsymbol{J}_{e_j}(\boldsymbol{r}, t) \rangle = \sigma k T \tau^{-1} \delta_{ij}. \tag{11}$$

The time dependence of the correlation function can be deduced from the evolution equation of $\delta \boldsymbol{J}_e$ obtained from (5), i.e.

$$(\delta \boldsymbol{J}_e)^{\cdot}_i = -\tau^{-1} \delta \boldsymbol{J}_{e_i}, \tag{12}$$

and it is given by (Landau and Lifshitz 1967)

$$\langle \delta \boldsymbol{J}_{e_i}(\boldsymbol{r}, t) \delta \boldsymbol{J}_{e_j}(\boldsymbol{r}, t+t') \rangle = \sigma k T \tau^{-1} \delta_{ij} \exp(-|t'|\tau^{-1}). \tag{13}$$

In the limit when the relaxation time τ tends to zero, equation (13) leads to

$$\langle \delta \boldsymbol{J}_{e_i}(\boldsymbol{r}, t) \delta \boldsymbol{J}_{e_j}(\boldsymbol{r}, t+t') \rangle = 2\sigma k T \delta_{ij} \delta(t') \tag{14}$$

in accordance with the classical expressions of the Nyquist theorem (Landau and Lifshitz 1969).

While the generalised Gibbs equation (1) has received a great deal of attention, both from a macroscopic and a microscopic point of view (Müller 1967, Lambermont and Lebon 1973, Lebon 1978, Jou *et al* 1979 and Lebon *et al* 1979), and the extended Ohm equation (5) has been widely used in a broad range of problems (Spitzer 1956), the hypothesis (8) is rather new. Here we have shown that it is able to describe the fluctuations of the electric current in a rigid conductor, and to reproduce as a limit case such a classical and well known result as the Nyquist theorem. It may therefore be of interest to gain some further insight into the validity and significance of the new relations established by hypothesis (8).

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